Mr. Bo
Alg 9 Notes

Name _____
Date _____

Solving Quadratic Equations by Completing the Square

Problem 1: When Factoring Fails

Consider the quadratic equation:

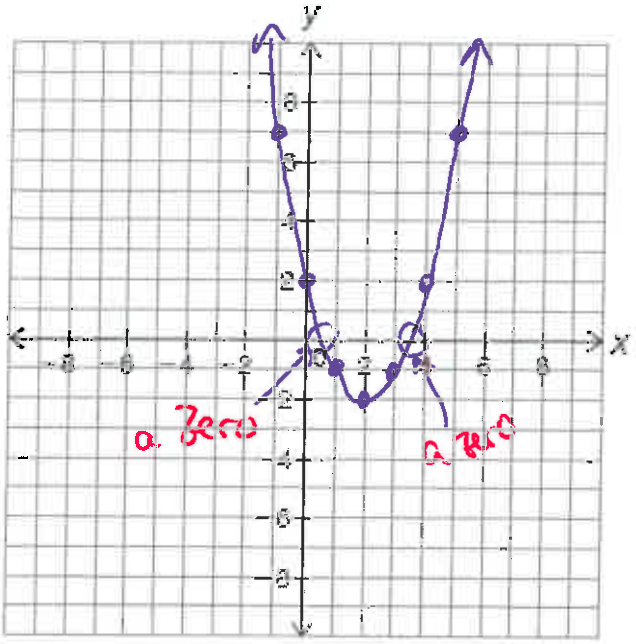
$$x^2 - 4x + 2 = 0$$

Explain why the roots of the quadratic can't be found by factoring.

factors of 2, which are 2 and 1, will not added to -4.
the trinomial can therefore not factor.

Does this mean it does not have zeros? No, it may still have zeros.

Use the TABLE feature of your calculator to graph the quadratic on the grid below.



x	y
-1	7
0	2
1	-1
2	-2
3	-1
4	2
5	7

Does the function have zeros? What are they, approximately? Explain your answer.

the function has 2 zeros, where the graph crosses the x-axis.

they are $x \approx 0.5$ and $x \approx 3.4$.

Problem 2 - Completing the Square

Completing the square is a method for finding the roots of a quadratic.

Basic Steps:

1. Create a perfect square trinomial of the form: $(x+a)(x+a)$ (identical factors.)
2. Rewrite the perfect square trinomial in the form: $(x+a)^2$
3. Solve the resulting expression by: taking square roots.

A perfect square has 2 identical factors.
ie) $3 \cdot 3 = 9$ (identical factors; perfect square)

Detailed Steps: **Example:** $x^2 - 4x + 2 = 0$

1. Move the constant term, +2, to the other side of the = sign.	$x^2 - 4x = -2$
2. Divide the middle term, -4, in half.	$\frac{-4}{2} = -2$
3. Square the result of step 2 and add it to both side of the = sign. **Hint:** it helps to write $(-2)^2$ rather than 4.	$(-2)^2 = 4$ $x^2 - 4x + 4 = -2 + 4$
4. Rewrite the left side of the equation as a *perfect square trinomial*. **Hint:** notice how the (-2) from step 2 is used inside the ( )'s.	$x^2 - 2x - 2x + 4 = 2$ $x(x-2) - 2(x-2) = 2$ $(x-2)(x-2) = 2$
5. Rewrite the perfect square trinomial in the form $(\quad)^2$.	$(x-2)^2 = 2$
6. Solve the resulting equation by taking square roots.	$x - 2 = \pm\sqrt{2}$ $x = 2 \pm \sqrt{2}$ $x = 2 + \sqrt{2} \approx 3.4$ $x = 2 - \sqrt{2} \approx 0.6$

Practice Examples:

1. $x^2 - 6x + 4 = 0$

$\frac{-6}{2} = -3$ $(-3)^2 = 9$

$x^2 - 6x + 9 = -4 + 9$

$x^2 - 3x - 3x + 9 = 5$

$x(x-3) - 3(x-3) = 5$

$(x-3)(x-3) = 5$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

2. $x^2 - 12x + 6 = 0$

$\frac{-12}{2} = -6$ $(-6)^2 = 36$

$x^2 - 12x + (-6)^2 = -6 + (-6)^2$

see hint about Step #3. This allows us to skip some of the steps.

$(x-6)(x-6) = 30$

$(x-6)^2 = 30$

$x - 6 = \pm\sqrt{30}$

$x = 6 \pm \sqrt{30}$

3. $x^2 + 8x - 9 = 0$

$\frac{8}{2} = 4$ $(4)^2 = 16$

$x^2 + 8x + (4)^2 = 9 + (4)^2$

$(x+4)(x+4) = 25$

$(x+4)^2 = 25$

$x + 4 = \pm\sqrt{25}$

$x + 4 = \pm 5$

$x = -4 \pm 5$

$x = -4 + 5 = 1$ $x = -4 - 5 = -9$

4. $x^2 + 20x + 36 = 0$

$\frac{20}{2} = 10$

$x^2 + 20x + (10)^2 = -36 + (10)^2$

$(x+10)(x+10) = 64$

$(x+10)^2 = 64$

$x + 10 = \pm 8$

$x = -10 \pm 8$

$x = -10 + 8 = -2$ $x = -10 - 8 = -18$

5. $x^2 + x - 10 = 0$

$x^2 + x + \left(\frac{1}{2}\right)^2 = 10 + \left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right) = 10.25$

$\left(x + \frac{1}{2}\right)^2 = 10.25$

$x + \frac{1}{2} = \pm\sqrt{10.25}$

$x = -\frac{1}{2} \pm \sqrt{10.25}$

6. $x^2 - 3x + 1 = 0$

$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = -1 + \left(-\frac{3}{2}\right)^2$

$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) = 1.25$

$\left(x - \frac{3}{2}\right)^2 = 1.25$

$x - \frac{3}{2} = \pm\sqrt{1.25}$

$x = \frac{3}{2} \pm \sqrt{1.25}$